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ERRATUM

Strategy-proof cardinal decision schemes

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As was first pointed out to us by John Hegeman,[1] the proof of Claim 3, Theorem 1, in Dutta et al. (2007) is not correct, since it is based on interchanging two limits which is not justified without, for instance, a continuity assumption.

In this note we first give an alternative proof of $\lambda_j \leq \lambda'_j$ (notations as in Dutta et al. (2007)).

To show this assume, to the contrary, $\lambda_j > \lambda'_j$. By Claim 2 we can take u_1 with $\tau(u_1) = a_j$ and—for simplicity—$u_1(a) = 0$ for all $a \neq a_j$. Let $0 < \varepsilon < \frac{1}{2}(\lambda_j - \lambda'_j)$ and let η_2 be so small that $|\varphi_j(u_1, u_{kj}^{\eta_2}) - \lambda'_j| < \varepsilon$. Observe that agent 1's utility in this profile is equal to $\varphi_j(u_1, u_{kj}^{\eta_2})$.

[1] Stanford University.

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Next, take η_1 so small that $|\varphi_j(u_{jk}^{\eta_1}, u_{kj}^{\eta_2}) - \lambda_j| < \varepsilon$. This is possible in view of Claim 2 (with the roles of the agents there reversed). Then according to u_1 agent 1's utility is now $\varphi_j(u_{jk}^{\eta_1}, u_{kj}^{\eta_2})$, but $\varphi_j(u_{jk}^{\eta_1}, u_{kj}^{\eta_2}) > \varphi_j(u_1, u_{kj}^{\eta_2})$. This violates strategy-proofness.

Unfortunately, at this moment we do not know how to prove the reverse inequality $\lambda_j \geq \lambda'_j$ and, thus, Claim 3, without making additional assumptions. One possibility would be to extend the set of admissible utility functions by dropping the requirement that there be a unique top alternative and assume continuity of the **CDS** φ. Another possibility is to strengthen the unanimity condition by requiring that if every agent in a preference profile has the same two top alternatives, then all other alternatives should receive zero probability. A third possibility is to impose, additionally, the following requirement on φ, which is a kind of unanimity:

($*$) For all admissible profiles u and all $a_j \in A$, if $u_i(a_k) \geq u_i(a_j)$ for all $a_k \in A \backslash \{a_i\}$ and $i \in N$, then $\varphi_j(u) = 0$.

In other words, if the agents have a common bottom alternative, then that alternative should receive zero probability. We will prove $\lambda_j \geq \lambda'_j$ under this additional assumption ($*$). To the contrary, assume $\lambda_j < \lambda'_j$. Let $0 < \varepsilon < \frac{1}{2}(\lambda'_j - \lambda_j)$. By Claim 2, we may assume that the utility functions under consideration have a common bottom alternative b. Also by Claim 2, we may assume that $u_{jk}^{\eta_1}$ satisfies $u_{jk}^{\eta_1}(a) = 1 - \eta_1 - \alpha(\eta_1)$ for all (if any) $a \neq a_j, a_k, b$, with $\alpha(\eta_1) > 0$ satisfying

$$\frac{\eta_1 + \alpha(\eta_1)}{\eta_1} < \frac{1 - \lambda_j - \varepsilon}{1 - \lambda'_j + \varepsilon} \tag{1}$$

(observe that the latter expression is greater than 1).

Now let u_1 with $\tau(u_1) = a_j$ and let $\eta_2 > 0$ so small that $|\varphi_j(u_1, u_{kj}^{\eta_2}) - \lambda'_j| < \varepsilon$. Next, let $\eta_1 > 0$ so small that $|\varphi_j(u_{jk}^{\eta_1}, u_{kj}^{\eta_2}) - \lambda_j| < \varepsilon$. Then player 1's utility (according to $u_{jk}^{\eta_1}$) from truthful reporting is smaller than

$$(\lambda_j + \varepsilon) + (1 - \lambda_j - \varepsilon)(1 - \eta_1) \tag{2}$$

whereas his utility from falsely reporting u_1 is greater than

$$(\lambda'_j - \varepsilon) + (1 - \lambda'_j - \varepsilon)(1 - \eta_1 - \alpha(\eta_1)). \tag{3}$$

Note that for (3) we have used that the bottom alternative b receives zero probability by ($*$). By (1) it follows that the expression in (3) is strictly greater than the expression in (2), which violates strategy-proofness.

At the moment we cannot show that ($*$) nor any other of the mentioned alternative additional assumptions are redundant.

Reference

Dutta B, Peters H, Sen A (2007) Stategy-proof cardinal decision schemes. Soc Choice Welfare 28: 163–197